THE TEACHING OF MATHEMATICS IN THE JUNIOR HIGH SCHOOL.

By Margaret Elsie Davis.

The phase of the J. H. S. movement which is of vital and immediate interest to us is the teaching of mathematics in this school unit. However, the problems that require consideration in connection with this branch of the curriculum are necessarily related to the problems of the whole movement and must be solved with due regard for that relation. Among the problems to which I refer are those of aim, subject-matter, method, sequence of subjects and the time element.

The present attitude toward and conception of these problems have been in the forming since the later eighties at least. They are the "result of dissatisfaction with (1) the existing rigidity of the grade system, (2) the lack of economy in education, (3) the failure to relate school work to social and economic conditions of life and (4) the failure to recognize and to provide for the changing conditions in the physical and mental characteristics of those to be educated, particularly during the period of adolescence."

To overcome these defects the type of school known as the Junior High School has been evolved. An examination of the ideas of the J. H. S. which are held by the most prominent educators shows that the "principle of individual differences is emphasized more than any other; but other principles regarded as vital are brought out: namely, the reorganization of subject-matter from a social standpoint and its placement upon a sound psychological and pedagogical basis for instruction."

These and minor principles when applied to and executed in the mathematics course of the J. H. S. should result in the following conditions:

Mathematics work

(1) which is suitable for all pupils approximately 12 through 16 years of age

(*a*) who will remain in school and pursue elective mathematics in the Senior H. S., and

(*b*) who for various reasons will assuredly leave school early, and who ought to be given some insight into mathematics of a varied and a vocational character;

(2) which reveals the pupils' interests, aptitudes and capacities by the use of mathematical material in itself worth while;

(3) which open up the mathematical possibilities in the senior subjects and in several industries of local importance;

(4) which provides for individual mathematical differences by differentiated curricula and by the organization of groups homogeneous in ability;

(5) which uses methods intermediate between those of the elementary and the high school;

(6) which provides for specially prepared teachers of mathematics through the partial or full use of the departmental plan;

(7) which uses subject-matter that will meet immediate and certain assured future needs along social and economic lines.

Subject-matter of the Mathematics Course as a Whole.

One of the best general statements regarding the principles underlying the reorganization of the mathematics course of J. H. years was presented by the National Society for the Study of Education in the 15th Yearbook—Part III. It specifies that in the place of arithmetic, algebra and geometry, which represent a logical and not a psychological sequence, a course in mathematics should be substituted which represents a unification of these three subjects after certain parts of each have been eliminated. Arithmetic will perhaps form the bulk of instruction in the seventh and the first half of the eighth grade. The last half of the eighth year and the first part of the ninth will consist mainly of algebra, with special emphasis upon the equation, but at the same time the facts of geometry applicable to the work in algebra could well be given. Toward the end of the ninth year the work will be mainly geometry.

A study of the various recommendations of educators and of course of study indicates a general tendency to incorporate in the body of subject-matter,

(1) arithmetic (largely of the social and economic type),
(2) experimental or constructional geometry,
(3) algebra,
(4) trigonometry (elements only),
(5) graphs,
(6) demonstrative geometry (variable from none to one-half year's work).

Several prominent educators favor differentiated courses, *e. g.*, shop and business mathematics, in the ninth year.

METHODS OF THE MATHEMATICS COURSE AS A WHOLE.

The aims and principles, previously stated, should determine to a very great extent the method, and therefore, the vitality and appeal of the course.

The methods of the past against which a reaction has occurred were logical rather than psychological. To a large extent they were systematic. Topics were introduced by an elaborate treatment of definitions and rules which meant little or nothing to pupils because they did not arise through a need for them nor was there any immediate application of them. Naturally very little interest accompanied such methods which lacked meaning for the pupils.

Insofar as the technique of past methods was good and at the same time made it possible to arouse interest and to provide motive and application in its desirable to retain it.

The project method, so widely advocated and used at the present time, develops the subject in connection with carefully chosen problems. It emphasizes the whole-hearted purposing of the doer and thereby involves the best of the modern tendencies in education. It is excellent as a means of arousing thought and preparing a student to solve the mathematical difficulties of life experiences. However, if used exclusively, it is likely to prove too slow. I may exclude the necessary drill which is so essential.

The method should be dominated by understanding, appreciation and skill. New principles should be introduced by means

of simple, concrete exercises and be consciously developed. The processes should be treated rationally. The individual should observe and think for himself. The presentation should be followed by drill exercises to secure a satisfactory degree of skill and by suitable applications to insure appreciation of the principle being taught.

A moderate amount of supervised study is desirable in order that pupils may have the advantage which such work offers, particularly that of meeting individual needs.

Correlation of the right kind should be employed. The various branches of mathematics are not to be taught independently of one another. The early introduction of geometry is a great aid in making clear algebraic processes and principles. As far as possible all the important facts and principles of mathematics should be kept under review in successive, related topics, to insure a greater skill in the use of them and an increased probability of their permanent retention. Correlation should be understood to mean unification and clarification, not fusion, which so often results in confusion.

The mechanical juggling of symbols and figures is discouraged. The teaching should not be primarily a text-book affair. The J. H. S. calls for more mathematics and less text-book and symbolism.

The motivation needs to be of the strongest kind. The idea embodied in this term is very important. It is regarded by some educators as the chief hope of improvement in J. H. S. mathematics.

In general, it may be said that the method is such an important item that, through its improvement only will mathematics thrive and survive.

Subject-matter of Arithmetic.

The fundamental changes in subject-matter proposed for arithmetic are the elimination of or reduction of time to be given to certain topics and the increase of time or emphasis to be given to others because of their social interest.

Mr. W. A. Jessup has prepared a table based upon a questionnaire sent to about 1,700 city superintendents and to every sixth county superintendent in the U. S. The majority of these

superintendents favor elimination of or less time to be given to apothecaries' and Troy weight, such measures as furlong, rood, dram, surveyors' tables, foreign money, reductions of more than two steps, G.C.D., L.C.M., true discount, cube root, partnership, compound proportion, compound and complex fractions, certain cases in percentage, annual interest, longitude and time, unreal fractions, metric system, progression and aliquot parts.

They favor more attention to the fundamental processes, fractions, percentage, interest, saving and loaning, banking, borrowing, building and loan associations, investments, stocks and bonds, taxes, levies, public expenditure, profits and utilities.

Closely allied with this question of elimination is that of the time to be devoted to arithmetic. An examination of various curricula shows an average difference of about one half year. *E. g.*, the National Society for the Study of Education places arithmetic in the seventh and first half of the eighth grade; the Rochester plan assigns it to the seventh year; and the ethical culture plan devotes only part of the seventh year to arithmetic and the remainder to intuitive geometry.

The topics for which more time is advocated by the superintendents to whom reference was made are fairly representative of those holding a place in the majority of the J. H. S. curricula.

Methods of Arithmetic.

The arithmetic work assigned to the J. H. S. involves practically no new processes and principles. The new work is largely informational in character and its source lies in the social and business conditions indicated by the topics outlined in the subject-matter.

The applications of percentage should be introduced by means of parallel types of problems in common and decimal fractions and percentage.

Thorough reviews and drills in the fundamental processes, fractions and decimals should occur as a consequence of this method of treating the new work. Also, the fundamental unity of arithmetic is emphasized, the work is made clearer, and time is economized.

The arithmetic must be correlated with the lives of the pupils

by finding genuine applications of the work that are really within their experience and life activities.

As an aid in analysis the unknown term and the equation from algebra are introduced wherever needed.

Mensuration and other topics of measurement should be facilitated by the introduction of constructional geometry.

Constructional, Observational, or Intuitive Geometry.

Mr. J. C. Brown's summary of the world's curricula in mathematics as outlined in reports of the International Commission states: "Some instruction in constructional, observational or intuitive geometry is always offered during the sixth, seventh, and eighth school years. This instruction is always of a propædeutic nature. Much emphasis is placed upon estimates and constructions.

"In all of the schools of Europe algebra and geometry are studied simultaneously during a considerable number of years. The various mathematical subjects are more closely correlated than in this country. A pupil who is studying geometry can use his arithmetic and algebra more readily than is the case with the average American boy. The introduction of the trigonometric functions while the pupil is studying similar figures in geometry has the sanction of most of the best teachers abroad. The distinction between plane and solid geometry is much less marked than in this country. This is due, in part at least, to the fact that models are very extensively used in the study of geometry."

The consensus of opinion of mathematical educators is that geometry should be begun as early as possible and certainly before algebra. Mr. William Betz maintains that it should be possible in the course of time to provide for some geometric instruction in the seventh grade, where it belongs.

Historically, this sequence, *i. e.*, arithmetic, geometry, algebra is the only correct one and experience verifies this.

Intuitional or constructive geometry is more concrete than algebra and admits of more simple illustration.

The algebraic formula may be gradually introduced in connection wtih the geometry work, so that the student may be made aware of the value of algebra as a working tool before he proceeds to the study of algebra.

SUBJECT-MATTER OF GEOMETRY.

The subject-matter of geometry may be grouped under the geometry of (1) form, (2) size, and (3) position. The majority of courses include part of all of the following:

1. Historical introduction.
2. Classification of forms.
2. Study of
   lines,
   angles,
   triangles,
   other polygons,
   circles.
4. Use of drawing instruments:
   slide rule,
   compass,
   protractor,
   right triangle,
   ruler.
5. Construction of
   angles,
   triangles,
   perpendiculars.
6. Bisecting
   lines,
   angles.
7. Parallels.
8. Drawing to scale.
9. Accurate proportions.
10. Congruence.
11. Equality.
12. Similarity.
13. Symmetry.
14. Geometric measurements:
    lines,
    surfaces,
    solids.
15. Simple surveying.
16. Ratio and proportion in connection with similar figures.
17. Locating points by lines and angles.

Methods of Geometry.

General principles of method were stated for the entire course in mathematics. A few details of the method follow.

The work at first is entirely observational and intuitive, but by low degrees rational processes are introduced.

Thoroughness of treatment is not sacrificed to breadth of treatment.

In whatever grade the geometry is begun it is advisable to give a preview of the course lasting two or three weeks. The pupils should get an idea of what geometry is, not through a dissertational course but through a concrete introduction. The geometry should not begin with projects but rather with evolutionary views.

Pupils should be shown how geometry originated through measurement as arithmetic did through counting. (McLellan and Dewey give a thorough treatment of this in "Psychology of Number.")

Projects may be brought in by asking, "Why did people count? Why did they measure?" Through these projects the geometry involved in (1) shelter, (2) clothing and (3) food may be discussed.

1. Shelter: Refer to the building of primitive houses, *e. g.*, the hemispherical home of the Esquimaux and the form of the Indian hut. Use these forms in the room and lead the children to see that through the activity of house building we get fundamental forms.

2. Clothing: Present looms used in weaving. Notice that the warp and woof give parallel lines, squares, etc.

3. Food: The primitive activity of agriculture gave geometric forms. Ask why so many fields are rectangular, why the rectangle is the form of practical geometry and the triangle of theoretical geometry.

Show examples of ornamental tatooing, Indian basketry and weaving.

The purpose of such a course is (1) to interest pupils in forms, (2) to get pupils to draw forms, and (3) to develop geometric ideas.

Through the study of ornamental objects lead the pupils to an appreciation of the four principal elements of geometry, *i. e.*,

(1) equal spacing, (2) symmetry, (3) congruence, and (4) similarity.

Lead the pupils to discuss the school room as a laboratory for geometry. Ask why the room, doors, windows, etc., are rectangular and the subsidiary forms have the same shape as the principal form of the room. Parallel lines exist and are important everywhere. Lead the pupils to look for them in nature, art and industry.

Note the symmetry of the butterfly and other natural objects.

Encourage pupils to prepare a geometry museum as an equipment to motivate the work.

Use pictures that represent designs of buildings.

Use slides loaned by the State Department of Visual Instruction to illustrate geometrical forms seen in the Egyptian structures, the Parthenon, etc.

Visit a house of industrial photography.

Have geometric exhibits in school.

Such exercises as the above permit of "geometry's working its way into children, since it cannot be taught into them."

Algebra Subject-matter.

This includes practically all of the topics usually treated in elementary algebra, *i. e.,*

1. Notation,
2. Symbols,
3. Formulas,
4. Equations,
5. Graphs,
6. Negative numbers,
7. Addition, subtraction, multiplication and division,
8. Fractions,
9. Factoring,
10. Equations, simple and quadratics,
11. Methods of checking,
12. Square root and Pythagorean theorem.

Pupils should be familiar with some of this subject-matter through its use in arithmetic and geometry. They may have some idea of (1) the nature of algebra and of its importance through their work with formulas, graphs and equations, (2)

the use of negative number, and (3) the applications of algebra to measurements of various kinds and in the solution of arithmetical problems.

Methods of Algebra.

In the past algebra has been taught altogether too often as a symbol-juggling course. Needless to say there must be a decided reform along this line in order that children may catch the spirit of the subject. The pupils in the J. H. S. algebra classes represent future artists, carpenters, some Senior H. S. and college candidates, etc. It would not permit the J. H. S. to fulfill its function if algebraic methods were such as to drill all the pupils until they attained "professional proficiency in factoring unusual expressions, removing ingenious nests of parentheses and simplifying a marvelous complexity of radicals."

The method should emphasize algebra as the shorthand of science. *E. g.*, consider the statement of the area of a rectangle. The generalization in longhand form is the complete sentence, "The area of a rectangle equals the product of the length and the width." But since it is a fundamental tendency of the mind to economize we may "economize" the above statement thus, $A = lw$.

The formula work may be motivated by a discussion of the ways in which we economize by shorthand in life, *e. g.*, commercial shorthand uses the initials U. S. A., C. O. D., a person's own initials.

The pupils may be asked to name various kinds of quantities that involve counting and measuring and to list these according to the branch of industry from which they come, *e. g.*,

Businesses	*Mensuration*	*Science*
principal	length	watt
percentage	width	pressure
interest	height	resistance
discount	distance	ohm
profit	radius	ampere
loss	circumference	

The field in which the pupils will probably find the greatest number of words is mensuration. This field is good for illustration of the point, *i. e.*, shorthand expression of terms by use

of initials, since it affords opportunity for *showing* things and makes a real background to which to attach symbolism.

Point out the use of shorthand for expressing the measurement of geometric figures, *e. g.,* $4s$ (sides) expresses the perimeter of the square.

The method should show clearly that algebra is a tool for solving applied problems. This is done through a study of the equation. Ask the pupils to consider the following problem. A coal bin, large enough to contain 8 tons of coal, is to be built in a certain cellar. The arrangement of the cellar requires the bin to be 6 feet wide and 6 feet high. What must be the length of the bin, allowing 36 cu. ft. for 1 ton of coal? First ask for a solution by use of the complete anaylsis method. Then ask for the formula for volume, and from this the equation which becomes $6.6.x = 288$, giving x a value of 8.

The superiority of the equation method becomes so apparent that pupils readily grasp its value.

Since the representation of generalizations by algebraic equations is considered as one of the most unique contributions to civilization, we cannot afford to neglect its emphasis in algebra.

The method of algebra should present clearly the function of the graph as a superior means of presenting statistics, etc. Contrast this with the table and descriptive methods of presenting facts, to make the advantage of the graphical method prominent.

Lastly, let me refer to the need of unifying all the mathematics work by correlating the algebra wherever possible with the arithmetic and geometry which precede it.

Trigonometry.

Many of the newer courses of study in mathematics include an introduction to the meaning of and the practical uses of trigonometry.

The subject is introduced with the tangent—the simplest and most natural trigonometric function, because of the variety and nature of its applications.

The use of the transit for measuring angles in outdoor work is advised.

There is a modern tendency to develop the idea of function in connection with the arithmetic work since problems in arith-

metic illustrate the great importance of functions in mathematics.

Because mathematics is so largely a science of function it is considered advisable to introduce this work earlier in the course. The name "function" and the definition of the term should be postponed until the idea is thoroughly established.

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